The firm's objective function as a collective choice problem*

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In standard price theory, firms are assumed to maximize profits. There exists a well-established literature that discusses the empirical relevance of this assumption, and implications of possible alternative objective functions.[1] More recently, the question of the firm's objective function has arisen in the context of markets under uncertainty, and there have been a number of claims that there is no general, obvious, objective function under such conditions.[2] Also, in attempts to model monopoly and monopolistic competition in a Walrasian system, it has become apparent that there is some difficulty with the objective function of the firm.[3] The purpose of this paper is to examine the firm's problem as a collective choice problem and demonstrate that the difficulties encountered in the monopolistic competition, and uncertainty literatures are, in essence, the same as those discussed in the public good – externality literature. The central result will be a simple application of Arrow's (1963) Possibility Theorem demonstrating that, in general, the firm's shareholder's preferences cannot be aggregated into a constitution for the firm. Of course, there are special cases where constitutions exist for firms, but I will demonstrate why they are degenerate cases of the Arrow theorem.[4]

The paper proceeds as follows: in Section 1 I have sketched the basic theoretic structure of the shareholder-firm problem; in Section 2, Arrow's Possibility Theorem is applied to the model of Section 1; in Section 3 I have examined some degenerate cases where firm constitutions do exist; in Section 4, I have applied the results of Sections 2 and 3 to a number of seemingly diverse problems in the literature; and finally in Section 5 there are some comments on the relationship between the constitution problem and incentive compatible mechanisms.

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1. The shareholder-firm model

The purpose of this section is to set out a general structure that can be applied to a wide variety of market models, that include competitive, non-competitive and very general externality arguments.

Imagine we have an economy made up of consumers and firms. Firms earn profits that are distributed to shareholders. Also there are possible externalities, or non-traded goods and services flowing between firms and consumers. Given this general structure, consider the set of feasible production plans for all firms in the economy. Assume that for each vector of production plans for all the firms, consumers will act in a competitive fashion, so that a unique equilibrium price vector will be determined. As part of that equilibrium one can deduce a vector of consumer indirect utility functions dependent upon the price vector, and the vector of firm production plans. Because the price vector is a unique function of the (parametric) production plans, the consumers have induced (or indirect) utilities over the production plans.

Formally, consider a set of consumers $I = \{1, \ldots, m\}$. Consider $i \in I$ to have a compact non-empty, consumption set $X_i \subset R^l$ such that $X_i = \tilde{X}_i \times Z_i$, where $\tilde{X}_i \subset R^{l_1}$, $Z_i \subset R^{l_2}$, and $l_1 + l_2 = l$. A consumption plan for consumer i is $x_i = (\tilde{x}_i, z_i) \in \tilde{X}_i \times Z_i = X_i$. The set $\tilde{X}_i$ is the consumption set of traded commodities; the set Z_i is the consumption set of non-traded commodities – or externalities.

Now assume:

A.1 Consumer i has a preference ordering[5] $\succcurlyeq_i$ defined over X_i, which satisfies:

For any $x_i' \in X_i$, the sets

$$\{x_i \in X_i | x_i \succcurlyeq_i x_i'\} \text{ and } \{x_i \in X_i | x_i \preccurlyeq_i x_i'\} \text{ are closed.}$$

Let there be a set of firms $J = \{1, \ldots, n\}$. Firm $j \in J$ is assumed to have a compact non-empty attainable production set $Y_j \subset R^{l_1}$, and a production plan $y_j \in Y_j$. Also let $y \in \prod_J Y_j = Y$.

Consumers and firms are assumed to trade in a market economy, which is not necessarily competitive.[6] In particular assume that consumer's actions do not affect market prices, but there is a possibility of market power associated with firm production plans. That is, consumers have certain knowledge of the impact of firms' actions on the set of relative prices in the economy. Also, we will assume that firms' production plans may affect consumer externality variables in the set Z_i. These assumptions can be formalized as follows.

B.1 Let there be a private ownership economy E such that:

(a) there is a set of possible prices $P = \{p \in R^{l_1}_+ | \sum_{k=1}^{l_1} p_k = 1\}$;

(b) the profit of the firm $j \in J$ is defined to be

$$\pi_j = py_j; \quad y_j \in Y_j; \quad p \in P$$

(c) the budget constraint for consumer i is

$$\{\tilde{x}_i \in \tilde{X}_i | p\tilde{x}_i \leq p\omega_i + \sum_{j \in J} \theta_{ij}\pi_j\},$$

where $\omega_i \in R^{l_1}$ is the endowment of consumer i;

and $\theta_{ij} \in \{\theta_{ij} \in R_+ | \sum_{i \in I} \theta_{ij} = 1\}$ for all $j \in J$.

B.2 There exists a function Λ: $\prod_{j \in J} Y_j \to P$. This function will be represented by the notation $p(y)$.

B.3 There exist functions Γ_i: $\prod_{j \in J} Y_j \to Z_i$, $\forall i \in I$. These functions will be represented by the notation $z_i(y)$, $i \in I$.

In summary then, consider the consumer i's choice problem as:

$$(1) \begin{cases} \text{Given } y' \in Y, \text{ then consumer } i \text{ chooses in the non-empty, compact set,} \\ X_i' \equiv \{x_i \in X_i | p(y')x_i \leq p(y')\omega_i + \sum_{j \in J} \theta_{ij} p(y')y_j'; \; x_i = (x_i, z_i(y'))\} \\ \text{a greatest element for his ordering } \trianglerighteq_i. \end{cases}$$

From theorem 4.6.1 in Debreu (1959), it follows that there exists a continuous utility function defined over X_i'. But X_i' is non-empty and compact, by assumption: therefore, by the Weierstrass theorem there exists an $x_i^\circ \in X_i'$ which maximizes utility (or equivalently – is a greatest element for the ordering $\trianglerighteq_i$).

Clearly the solution to problem (1), for arbitrary $y \in Y$, induces a preference ordering over the set Y for each consumer i. Therefore we have shown the following (trivial) lemma.

Lemma 1

Given assumptions A.1, B.1-B.3, there exists a preference ordering $\trianglerighteq_i'$ over the set Y, for all $i \in I$.

For future reference we will designate this set of states and individual preferences as $\{(\trianglerighteq_i'), Y\}$.

In summary then, we have derived a set of preferences over the production plans of firms in the economy. This derivation is sufficiently general to allow three main ways that a firm's production decision may affect a consumer's welfare. First, the consumer may be shareholder, sharing in the firm's profits; second, the consumer may be affected by production externalities produced by the firm; and finally the firm's production decision may be 'large enough' in the economy to alter the slope of the consumer's budget constraint through changing consumer prices.

2. A possibility theorem

Arrow (1963) has provided a powerful result showing that one cannot find, in general, a constitution (or collective choice function), that will aggregate the preferences of a group of agents. In this section we will outline Arrow's result and apply it to the collective choice problem derived in Section 1.

In constructing collective choice rules, Arrow imposed four conditions that he argued would be reasonable restrictions on a constitution. His result can be summarized as follows:[7]

Definition 1: A collective choice rule is a functional relation f such that, for any set of n individual orderings $\trianglerighteq_i, i \in I$, on X, one and only one social preference relation $\trianglerighteq_s$ is determined by $\trianglerighteq_s = f(\trianglerighteq_1, \ldots, \trianglerighteq_n)$.

Definition 2: A Social Welfare Function (SWF) (or a Constitution) is a collective choice rule f, the range of which is restricted to the set of orderings over the choice set, X.

Consider the conditions:

Condition U (Unrestricted domain)
The domain of the rule f must include all logically possible combinations of individual orderings over X.

Condition P (Pareto principle)
For any pair $x', x'' \in X$, $[\forall i \in I: x' \trianglerighteq_i x''] \Rightarrow x' \trianglerighteq_s x''$.

Condition I (Independence of irrelevant alternatives)
Consider $\{\trianglerighteq_i\}$ and $\{\trianglerighteq_i'\}$ and the derived social binary relations $\trianglerighteq_s = f(\trianglerighteq_1, \ldots, \trianglerighteq_n)$ and $\trianglerighteq_s' = f(\trianglerighteq_1', \ldots, \trianglerighteq_n')$. If for all $x', x'' \in A \subset X$, $x' \trianglerighteq_i x'' \Leftrightarrow x' \trianglerighteq_i' x''$ for all $i \in I$, then $C(A, \trianglerighteq_s) = C(A, \trianglerighteq_s')$.[8]

Condition D (Non-dictatorship)
There is no individual $i \in I$, where for all $x', x'' \in X$ such that $x' \mathbin{\textcircled{\scriptsize$\succeq$}}_i x''$ implies $x' \mathbin{\textcircled{\scriptsize$\succeq$}}_s x''$.

Arrow was able to prove the following powerful theorem.[9]

Theorem A

Given $n \geq 2$, and X contains at least three distinct elements, then there is no SWF satisfying conditions U, P, I and D.

From the discussion in Section 1, Theorem A applies directly to the generalized shareholder-firm problem $\{(\mathbin{\textcircled{\scriptsize$\succeq$}}'_i), Y\}$. That is, we have the following simple corollary to Arrow's Theorem.

Corollary A.1
Given the generalized shareholder-firm problem $\{(\mathbin{\textcircled{\scriptsize$\succeq$}}'_i, Y\}$, where $n \geq 2$ and there are distinct elements $y', y'', y''' \in Y$, then there is no SWF satisfying conditions U, P, I and D.

Of course, corollary A.1 is hardly surprising given the generality of the externality and non-competitive mappings. Indeed, we can sharpen corollary A.1, restricting the choice set to a particular firm and inquiring whether there is a constitution that can be constructed from the preferences of the shareholders of that firm. That is, consider the choice set to be $Y_{j'}, j' \in J$,[10] and the relevant set of consumers $I_{j'} = \{i \in I | \theta_{ij'} > 0\}$, so that the collective choice problem is reduced to $\{\mathbin{\textcircled{\scriptsize$\succeq$}}'_i, i \in I_{j'}), Y_{j'}\}$. Therefore, we have the following corollary:

Corollary A.2
Given the restricted shareholder-firm problem $\{(\mathbin{\textcircled{\scriptsize$\succeq$}}'_i, i \in I_{j'}), Y_{j'}\}$ where $n \geq 2$ and there are distinct elements $y'_{j'}, y''_{j'}, y'''_{j'} \in Y_{j'}$, then there is no SWF satisfying conditions U, P, I and D.

The results in corollaries A.1 and A.2 are very general and include a number of well-known problems from public economics. For example, consider the old international trade problem that gave rise to the Kaldor-Scitovsky compensation tests, and the subsequent discussions on collective choice rules. Clearly, one can consider the jth 'country' to take the place of the jth firm in the above analysis. Another example, is the standard public good analysis, where the jth firm is replaced by the jth public good production technology, and the public goods are distributed via the mappings Γ_i. These examples should convince the reader that the firm's problem and the government-public good problem fall into the *same* general collective choice framework. Traditionally, microeconomic theory and public economics have stressed different aspects of this collective choice problem, by making different (implicit) assumptions

about the mappings Λ and Γ^i. These cases, as well as some other well-known special cases, will be discussed in the next section.

Before passing to these straightforward special cases, I should point out that we could abandon the strong requirement that the SWF must be a transitive binary relation, and therefore admit majority rule procedures. This is an interesting avenue of research, but we will not explore it here.[11]

3. Some special cases

The implications of corollary A.2 are extremely damaging for any attempt to find an objective function for a firm when that objective function is a constitution and there is non-competitive behavior and/or externalities. Many writers have merely *asserted* an objective function (for example, expected profit maximization, or expected utility of profit maximization). Nevertheless, there are some theoretical special cases of the shareholder – firm problem where a well-defined objective function does exist. In this section, we will explore some of these cases and relate them to corollary A.2.

3.1 *Perfect markets and profit maximization*

Consider an economy where there is a competitive market economy and no externalities, i.e. a *perfect* market, where the mappings Λ and $\{\Gamma_i\}$ are trivial. Furthermore consider the stronger assumptions on the consumer:

A.2 Let X_i be convex and the ordering $\trianglerighteq_i$ satisfy:

(a) For any $x_i' \in X_i$, the set

$\{x_i \in X_i | x_i \trianglerighteq_i x_i'\}$ is convex:

(b) For any $x_i \in X_i$, $\exists x_i' \in X_i$ such that $x_i' \triangleright_i x_i$.

Now it is easy to prove the following theorem:

Theorem B

Given a competitive market economy; assumptions A.1 and A.2 on all consumers; and the existence of a profit-maximizing production plan for every firm $j \in J$; then every consumer will prefer (or at least be indifferent to) the profit maximizing plan, over any other production plan, for firm $j \in J$.

Proof: This theorem (sometimes called the Fisher Separation Theorem) was illustrated in the two good case by Hirshleifer (1958). For a general proof see Milne (1974).

This result is at once powerful and appealing in its simplicity. It demonstrates that the usual profit-maximization assumption of microeconomics is not an independent hypothesis, but, under the given assumptions, a derivative constitutional rule. What is of interest here is why, given the special structural assumptions, is there a simple constitution for the firm? The answer is relatively simple: in the competitive market model it is easy to show that every consumer desires more wealth to less. Therefore, because profits enter as a component of a consumer's wealth (or not at all if he is not a shareholder), and there are no externalities, every shareholder will wish his partly, or wholly-owned firm to maximize profit.

In a perfect market, decisions are decided by derived preferences defined over the scalar wealth. But this is precisely the case – the case of a 'one commodity world' – where Arrow admits the possibility of the formation of a constitution.[12] The Arrow theorem A, is inadmissable, because the derived preference orderings $\trianglerighteq_i'$ do not satisfy the assumption of unrestricted domain, i.e., condition U (the only admissible preference orderings are identical!).

3.2 *One owner, or identical owners*

One simple way around the dilemmas posed by correlaries A.1 and A.2, is to restrict the number of shareholders to a single owner. With a single owner, the owner's preferences become the firm's constitution. A trivial generalization of this assumption occurs with m identical owners, where identity is translated as identical consumption sets, preferences, endowments and shareholding proportions.

Of course, these assumptions were recognized by Arrow as simple degenerate cases of collective choice. That is, the case of a single owner violates the assumption that $n \geq 2$; and that of identical owners violates the assumption of Unrestricted Domain.

A slight variant on the identical owners condition arises when all consumers have identical homethetic preferences; there are no externalities (i.e., the maps Γ_i, $\forall i$ are trivial); and all consumers have the same endowments and shareholdings up to a positive factor of proportionality.[13] Because each consumer's problem is identical, except for an irrelevant scale factor (as far as choice of Y is concerned), there is a trivial unanimity constitution for the firm.

Observe that in this single owner (or identical owner) case, with non-trivial mappings Λ, Γ_i, it is easy to construct examples where the owner does not want his wholly-owned firm to maximize profit. Indeed, one can show that profit is not defined independently of the price normalization rule (see Gabszewicz and Vial, 1972). Thus, in the general case, the firm's production plan is influenced directly by the consumer's preferences, and the Fisher Separation Theorem fails.

Finally we might conjecture that the owner of a monopolistic firm should

price discriminate in favour of himself.[14] This is consistent with our model, by allowing the output of the firm to be sold in personalized markets. That is, the firm can allocate its output to different consumers and charge them different prices. Each personalized market is considered another commodity market. Using this device, the owner can discriminate in favour of himself in selling on his own commodity market. An alternative (but equivalent) method of modelling this behaviour is to treat the owner's consumption of his firm's output as an externality (or non-traded good). The discriminating price will be derived as the shadow price of the externality, from the owner's general consumption-production optimization problem.

3.3 *Separable environment*

Another approach is to restrict the price and externality mappings so that profit maximization reappears as the firm's constitution even though there remains some monopoly power and externality effects. I suspect such an idea is implicit in most partial equilibrium treatments of monopoly and external effects, that assume profit maximization. Let me sketch an argument that will produce such a result.

Consider a firm $j' \in J$. Assume that there are no externalities flowing from firm j' to the shareholders of firm j'. Furthermore, assume that the actions of firm j' and the actions of its shareholder-consumers do not affect the *relative* prices of goods that are relevant for the shareholder-consumers' opportunity sets; and assume that consumers $i \in I_{j'}$ have the profit of firm j' as their only source of profits. That is, formally:

C.1 There exists a partition of R^{l_1} into R' and R'', such that $\omega_i, X_i \subset R'$ for all $i \in I_{j'}$; and $Y_{j'} \subset R''$.

C.2 Let $P \equiv P' \times P''$ where P', P'' are price simpleces in R' and R'' respectively. Assume that Λ_j: $Y_j \to P' \times P''$ takes the particular form $(p', p'') = (\bar{p}, p''(Y_j))$, where $p', \bar{p} \in P'$, $p'' \in P''$ and $\bar{p}$ is a constant.[15]

C.3 Γ_i: $Y_{j'} \to Z_i$, for $i \in I_{j'}$, is a constant function.

C.4 For $i \in I_{j'}$, $\theta_{ij} = 0$ for $j \neq j'$.

Corollary B.1

Given a market economy, i.e. assumptions B.1-B.3; assumptions A.1 and A.2 on all consumers $i \in I_{j'}$; assumptions C.1-C.3 for $j' \in J$; and the existence of a profit maximizing production plan for firm $j' \in J$; then every consumer $i \in I_{j'}$ will prefer the profit maximizing plan, over any other plan, for firm $j' \in J$.

This result can be illustrated with a simple intuitive example. Imagine an Australian who is the sole owner of a flour mill in Germany. As an absentee owner, the Australian will not suffer, or benefit, from any obvious externalities. Any changes in German relative prices – resulting from the flour mill's

monopoly power – will effect the Australian owner only through the direct wealth effect from his earnings of German Marks.

I suspect that some variant of the above argument lies behind the defence of profit maximization in the standard microeconomic literature.[16] For *many* practical problems, this solution may be a reasonable approximation for predictive purposes.

4. Applications in the literature

In this section, I want to show that the problem of the firm's objective function has appeared in various guises in the economics literature. Although the fundamental collective choice problem is identical in the cases to be discussed, some authors appear to be unaware that they are constructing special cases of a more general collective choice problem.

4.1 *Investment and finance theory under uncertainty*

In corporate finance theory (and in the more general theory of production under uncertainty), many writers have asserted that there is no obvious objective function for the firm. Some writers impose the condition of a single owner, or a managerial dictator, so that the firm maximizes the expected utility of the sole-owner-dictator (this is the degenerate solution of subsection 3.2 above).[17] But since the important contribution of Diamond (1967), there has been a growing literature investigating more sophisticated objective functions for the firm under uncertainty. Diamond, in a stock market model, observed that if the firm's production function was of a special multiplicative type, then there existed a profit (or value) maximizing objective function for the firm: but without this restriction, there appeared to be no obvious firm objective function, and the Pareto Efficient conditions resembled standard Samuelsonian public good conditions. Subsequently, other writers have tackled the 'Diamond problem'.[18] The net result of these studies can be summarized as follows: if a firm issues securities (which are claims on net revenues in future periods), and these securities have perfect substitutes – in terms of stochastic returns – then the original owners of the firm will be unanimous in choosing the profit maximizing constitution. It has become commonplace to characterize the perfect substitutes assumption, by the alternative description, that the new security can be 'spanned' (in the sense that the new security's stochastic payoff can be obtained as a linear combination of existing securities' payoffs.) Unfortunately, 'spanning', has been interpreted as a very restrictive technical assumption, rather than the more usual economic argument of perfect substitutes in competitive markets.[19]

On the other hand, if the firm issues a new security (i.e., for which there is no

competitive substitute in returns), then the firm is acting as a monopolist, and in the general sense of corollary A.2 there is no constitution for the firm.

As one might expect, the creation of assets through corporate leverage, or changes in financial structure, raise the same problem: the Modigliani-Miller leverage proposition follows directly if the firm issues a bond which has a competitive perfect substitute; but if the bond is a 'new' security, the theorem fails, and there is a collective choice problem.[20]

Notice that in the investment, or financing decisions, there is a collective choice problem not only for the firm's original shareholders (i.e., corollary A.2), but also for the whole economy (i.e., corollary A.1). This intimate relation between the objective function for a firm and the more general problem of collective choice for an economy will be explored in the next section.

4.2 *Externality problems*

It is well known that externalities create difficulties for the standard market arguments. The non-existence of certain markets implies that the competitive allocation is not necessarily Pareto Efficient. But what is not widely appreciated, is our corollary A.2 – externalities and profit maximization may be inconsistent. For example, Arrow and Hahn (1971: Ch. 6) constructed a general equilibrium model with externalities, and, at the same time, they imposed profit maximization: these assumptions are not necessarily consistent.

The relationship between the objective function of the firm and externalities has been discussed in certain applied problems. For example, Drèze (1976) recognized that certain labour inputs to firms may involve externality problems, so that profit maximization may be an inappropriate constitution for Pareto Efficiency. Along similar lines, Arrow (1972) observed that a racially prejudiced owner of a firm will violate profit maximization, and the appropriate constitution for such a firm is maximization of the utility of the owner. Where there is one owner, as in this case, the constitution problem is the degenerate case of subsection 3.2; but where there are two or more owners, corollary A.2 demonstrates that no constitution exists.

One way around the externality problem is to introduce some variant of the Lindahl economy, where personalized markets are set up between agents producing externalities and agents receiving externalities.[21] In the competitive Lindahl equilibrium (if it exists, the externalities are fully 'priced out', and profit maximization (i.e., Theorem B) reappears as the appropriate constitution for any firm. One difficulty with this construction is the assumption of price-taking in the personalized markets. Because personalized markets imply small numbers, the competitive assumption is hard to justify.[22]

It is interesting to observe that the notion of cost-benefit analysis is the application of the profit-maximization objective to a public choice problem.

The method is to try and estimate Lindahl prices (in a rough and ready fashion) and then consider the public project as if it was a private project. In fact, some writers on government consider the government as a large firm, where the profits (rents) go to politicians or bureaucrats. Notice that for these arguments to apply, there must be competitive price-taking behaviour on the part of economic agents, and all the externalities must be 'priced out' in the Lindahl sense.

4.3 *Non-competitive models*

A number of attempts have been made to integrate non-competitive behaviour into a general equilibrium model. For example, Arrow and Hahn (1971: Ch. 6) contained an analysis of monopolistic competition; and Gabszewicz and Vial (1972) considered a general equilibrium oligopoly model. In both of these discussions, firms were assumed to maximize profit, but Gabszewicz and Vial realized that this assumption was difficult to justify.[23] In more recent contributions, Gevers (1974) and Hart (1976a, b) (in the context of stock-market models), have attempted to model non-competitive equilibria where firm's decisions are decided by voting constitutions. Gevers was able to show that one could obtain a Condorcet voting paradox with such a constitution: this result should not be surprising in the light of corollary A.2. Also, Hart attempted to model the decision problem by assuming a variety of voting processes, but he acknowledged that there is no guarantee that these equilibria exist or imply Pareto Optimality. Furthermore, Hart showed that in a suitably defined non-competitive model of the Gabszewicz and Vial type, that as the economy became 'large', a 'value maximization equilibrium' converges to a point in the core. All these results are consistent with corollary A.2 and Theorem B, where a natural constitution emerges in a large economy with agents unable to change the terms of trade through their own consumption or production decisions.

5. Collective choice and incentive compatibility

In a fundamental contribution, Satterthwaite (1975) (see also Gibbard, 1973), demonstrated, (a) the impossibility of a non-dictatorial, strategy-proof, voting procedure for a finite committee; and (b) the existence of a one-to-one correspondence between strategy-proof voting procedures and Arrow constitutions.[24] This second result, in conjunction with our corollary A.2, implies that there exists no strategy-proof voting procedure for the restricted shareholder problem.

In a previous paper, Hurwicz (1972) observed that in a *finite* agent, market exchange economy, the market system was not incentive-compatible (or in

Satterthwaite's terminology, strategy-proof). But, with a 'large' economy, the individual agents cannot alter the terms of trade, so that a 'large', competitive economy is incentive-compatible.[25] This observation is directly related to our Theorem B. In a competitive (i.e., 'large') economy, the Arrow theorem is degenerate, and profit maximization emerges as a constitution, which, by the Satterthwaite theorem, is a strategy-proof voting procedure.

Concluding comments

In this paper I have tried to make the simple point that the shareholder's problem is a collective choice problem. Because of Arrow's Possibility Theorem we must draw the pessimistic conclusion that under very general institutional arrangements there is no Arrow constitution for the firm. Of course, constitutions do exist for theoretical special cases, which have been widely used in the past literature. Although these cases have obvious important roles to play for many positive theoretical problems, they may be too simplistic to explain more complicated phenomenon relating to the financial and organizational structure of firms.

NOTES

1 See Marris (1964) and Williamson (1964).
2. For example, see Diamond (1967), Ekern and Wilson (1974), Radner (1974), Leland (1973, 1977), Drèze (1974) Gevers (1974), and Hart (1976a, b).
3. This issue has been discussed by Gabszewicz and Vial (1972).
4. The results in this paper appear to be implicit in many discussions in the literature, but I have been unable to find an explicit exposition. For example in Fama and Miller (1972, Ch. 2), the Arrow problem is mentioned, but not explored, to show its relationship with the profit maximization objective.
5. That is, a reflexive, transitive and complete binary relation (see Sen, 1970).
6. For example, our discussion will be consistent with the general model formulated by Gabszewicz and Vial (1972).
7. The terminology and statements are taken from Sen (1970, Ch. 3).
8. Let a binary relation R be defined on X; then define

$$C(X, R) = \{x \in X | xRy \text{ for every } y \in X\}.$$

9. See also the proof in Sen (1970: Ch. 3).
10. Implicitly, all other firms' production decisions are fixed, i.e.,

$$y_j = \bar{y}_j \quad \text{for all} \quad j \neq j'.$$

11. For a general discussion of this issue see Sen (1970) and the recent paper by Slutsky (1977). Some discussions of majority-rule shareholder decision rules are given in Gevers (1974) and Hart (1976), for the investment decision under uncertainty.

12. For an extended discussion, see Arrow (1963: Ch. 6, especially pp. 69-70).
13. If the consumers' utility functions satisfy the von Neumann-Morgenstern axioms then homotheticity implies that the utility functions must satisfy simple functional forms, i.e., the power, exponential and logarithmic functions (see Milne, 1979).
14. This suggestion was made by a referee.
15. In assumption C.2 I have used an explicit price normalization procedure: clearly other normalization rules could be used. For a good discussion of the importance and role of price-normalization assumptions in non-competitive models, see the discussion in Gabszewicz and Vial.
16. For example see Hirshleifer (1976: Ch. 9) for a verbal discussion.
17. This solution has been used to investigate standard microeconomic production decisions under uncertainty, for example Leland (1972); and also in investigations of the implications of adding uncertainty to international trade models; for example, see Turnovsky (1974).
18. For a sample of this literature see the references cited in note 2 above.
19. This argument is considered in more detail in Milne (1975, 1976a) and Grossman and Stiglitz (1977).
20. A detailed discussion of this problem, and the preceding literature is contained in Milne (1975).
21. Bergstrom (1976) has a very general formulation for this type of model. See also, his discussion of the limitations of the model; and his bibliography for previous discussions.
22. This observation is linked intimately with problems of incentive compatability. See the discussion in Section 5.
23. See Gabszewicz and Vial (1972: 394-396). Indeed there are other problems with the non-competitive models. For example, Roberts and Sonnenschein (1977) provide serious criticisms of continuity and convexity assumptions on reaction functions.
24. For brevity I have omitted strict definitions – see Satterthwaite (1975).
25. Hammond (1979) has provided a further investigation of the relationship between large economies and incentive compatability.

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